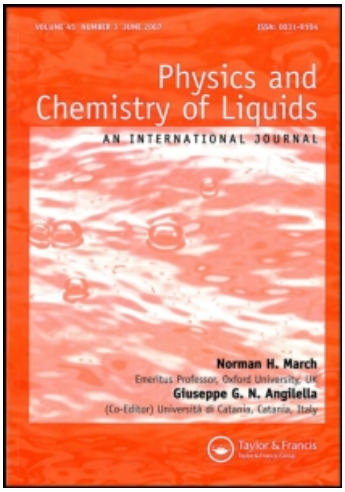

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Critical temperature for phase transitions in charged-particle assemblies

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Letter

Critical temperature for phase transitions in charged-particle assemblies

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A variety of phase transitions in both three- and two-dimensional assemblies of charged particles, with or without superfluidity, are compared and contrasted. Included here are (i) electron-hole droplets in five semiconductors, (ii) critical point properties of liquid alkali metals, (iii) transition temperature for non-*s*-wave pairing superconductors correlated with coherence length, and (iv) excitonic assemblies involving Bose–Einstein condensation.

Keywords: Phase transitions; Excitonic localization length

PACS: 64.60.-i; 03.75.Nt

1. Introduction

In earler work, Chapman and March [1,2] drew attention to regularities between the critical constants of the fluid alkali metals. In particular, they noted a correlation between critical temperature and critical number density, denoted by T_c and n_c respectively, of the form

$$T_c n_c^{-1/3} \cong \text{constant}. \tag{1}$$

Comparison of this result, characteristic of these "Coulombic" liquid metals, with the neutral fluids like neon and argon, showed that for the latter series the exponent $-1/3$ was to be replaced by $\approx +2$.

2. Electron-hole droplets compared with liquid alkali metals

Let us turn to compare and contrast the result (1) with the behavior of another fluid of charged particles, namely electron-hole droplets in semiconductors [3,4]. Recent interest

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was motivated by the time-resolved luminescence measurements of Shimano *et al.* [5] which allowed the study of an electron-hole liquid (EHL) in diamond, with a high critical temperature T_c of 165 K. While values of T_c are known now experimentally for at least five EHLs, the same is unfortunately not true for the critical density n_c. Therefore Leys *et al.* [6] studied, prompted by equation (1), these critical temperatures *versus* the known zero-temperature number density n_0 for diamond, Si, Ge, GaP, and 3C–SiC. They found an accurate fit of the data of the form (see figure 1 of [5])

$$T_c n_0^{-(1/2)} = 16.3 \times 10^{-9}\,\mathrm{cm}^{3/2}\,\mathrm{K} \tag{2}$$

to be contrasted with equation (1). We note that more than two decades ago, Reinecke and Ying [7] had anticipated a relation $T_c n_c^{-(1/2)} = \text{constant}$. If this latter result is combined with a "scaling law" $n_c/n_0 \cong 0.3$ proposed subsequently by Kalt *et al.* [8], then equation (2) follows qualitatively, as required by the experiment. We shall return to a relation of the form $T_c n_c^{-\kappa} = \text{constant}$ when we discuss quasi-two-dimensional excitonic assemblies subsequently.

3. Transition temperature for non-*s*-wave pairing superconductors correlated with coherence length and effective mass

We turn now to transitions which involve, in at least one phase, macroscopic manifestations of quantum mechanics. In this section, we consider the superconducting transition temperature T_c in non-*s*-wave pairing superconductors, and following Angilella *et al.* [9], specifically in (a) three-dimensional heavy fermion (HF) materials and (b) quasi-two-dimensional high-T_c cuprates.

The idea is to correlate the thermal energy $k_B T_c$ with what might be termed as a kinetic energy of localization, say ε, defined by [9]

$$\varepsilon = \frac{\hbar^2}{m^* \ell_c^2} \tag{3}$$

where m^* is the effective mass while ℓ_c is a characteristic length which remains to be chosen. Uemura *et al.* [10] had earlier recognized that m^* should enter inversely in determining the scale of $k_B T_c$. It then turns out that, with the rather natural choice of ℓ_c in equation (3) as the coherence length ξ, one can write for HF materials [9]:

$$k_B T_c = f_{HF}\left(\frac{\hbar^2}{m^* \xi^2}\right) \tag{4}$$

where f_{HF} goes from the linear form at small argument to constant at large argument. Some first-principles justification for such behavior is afforded by solution of the Bethe–Goldstone equation for the Cooper pairing problem by Angilella *et al.* [11]. A modification of $f(\varepsilon)$ in equation (4) is required for the quasi-two-dimensional high-T_c cuprates.

Having stressed here the importance, in such a quantal phase transition, of correlating the transition temperature T_c with a kinetic energy of localization as in equation (3), we turn finally to discuss the relevance of this to a different situation, namely Bose–Einstein condensation (BEC) in excitonic assemblies.

4. Bose–Einstein condensation in excitonic assemblies

Experiments [12–14] have by now been carried out on the BEC of excitons in two-dimensional assemblies created inside semiconductor quantum wells. It is also of interest here to note that such experiments have prompted a very recent theoretical contribution in this area by Crisan and Tifrea [15]. These authors construct a low energy model for the BEC in a quasi-two-dimensional excitonic gas, following earlier studies by Popov [16] and by Fisher and Hohenberg [17]. The later work of Pieri *et al*. [18] is also relevant in the present context. In particular, in [18] the transition temperature T_c is obtained from

$$k_B T_c = \frac{2\pi\hbar^2 n/m}{\left|\ln(\ln[1/na^2])\right|}, \tag{5}$$

where m denotes the Boson mass while a represents the range of the interaction between the (overall neutral) excitons.

It is now of interest to compare the RHS of equation (5) with the characteristic energy ε introduced in equation (3). This then allows the identification of the "localization length" for this BEC transition via

$$\ell_c^2 = \frac{\left|\ln(\ln[1/na^2])\right|}{2\pi n} \tag{6}$$

with n, as mentioned earlier, now being the areal density. If we neglect the weak dependence of the double-logarithm term in equation (6) on the dimensionless quantity na^2, then evidently $k_B T_c$ is proportional to $\hbar^2/(m\ell_c^2) \propto \hbar^2 n/m$, or

$$T_c n^{-1} \cong \text{constant}. \tag{7}$$

This is to be contrasted with equations (1) and (2) for admittedly very different phase transitions.

5. Conclusions

In summary, we have considered in this letter a variety of phase transitions ranging from the critical behavior of fluid alkali metals through electron-hole liquids (HLs) to specifically quantal transitions involving either superconductivity or superfluidity, both of which are macroscopic manifestations of quantum mechanics. In sections 2 and 4, regularities emerge relating critical temperature T_c and critical density n_c, while sections 3 and 4 are linked via a "localization length" ℓ_c.

While, in both three-dimensional heavy fermion (HF) assemblies and quasi-two-dimensional high-T_c cuprates, $k_B T_c$ correlates with a kinetic energy of localization, in which ℓ_c is essentially the coherence length ξ, in the BEC transition in quasi-two-dimensional assemblies, ℓ_c is dominantly proportional to $n^{-1/2}$ with n the areal density. However, residual dependence on the two-dimensional diluteness parameter $p = 1/\ln(1/na^2)$ with a the interaction range can alternatively be expressed in terms of the coherence length as given, for example, by Crisan and Tifrea [15], but we shall not pursue the detail further as this results in an implicit equation for T_c rather than the explicit form following from equations (3) and (6).

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